LORENTZ AND CPT VIOLATIONS FROM CHERN-SIMONS MODIFICATION OF QED

Alexander A. Andrianov

Instituto Nazionale di Fisica Nucleare Sezione di Bologna

Paola Giacconi & R. S.

Dipartimento di Fisica Università di Bologna

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0. INTRODUCTION

- **Lorentz symmetry breaking at the $M_{\rm Planck}$ scale :**

  ◦ $\mathcal{STRING}$ & $\mathcal{BRANE}$ $\mathcal{THEORIES}$

  ◦ $\mathcal{NON-COMMUTATIVE}$ $\mathcal{FIELD}$ $\mathcal{THEORIES}$

- **Induced low-energy very small deviation from the Lorentz covariant Standard Model :**

  ◦ CPT **violating modification of electrodynamics**

  `S.M. Carroll, G.B. Field & R. Jackiw (1990)`

  ◦ **Lorentz symmetry breaking from U(1) Anomaly**

  `A.A. Andrianov & R. Soldati (1995)`

  ◦ CPT **& Lorentz spontaneous breaking**

  `D. Colladay & V.A. Kostelecky (1998)`

  ◦ **explicit parity even Lorentz symmetry breaking**

  `S.R. Coleman & S.L. Glashow (1999)`

- **Lorentz non-covariant Lagrange models :**
  - ◦ **Power counting renormalizable**
  - ◦ $SU(3) \otimes SU(2) \otimes U(1)$ **gauge invariant**
- **Privileged Inertial Reference Frames (PIRFs) :**
  Cosmic Microwave Background (CMB) isotropic
  - ◦ **tiny anisotropies in laboratory experiments**
  - ◦ **translation invariance within PIRFs**
  - ◦ Lorentz symmetry breaking by **constant 4-vectors**
  - ◦ **Constant four-vectors** η_α & b_μ **very small** within **PIRFs** owing to **experimental bounds**
- η_α & b_μ may have **cosmological** origin :
  - ◦ $\mathcal{TORSION}$-like → $b_\mu = \epsilon_{\mu\nu\rho\sigma} \langle T^{\nu\rho\sigma} \rangle_{\underline{0}}$
  - ◦ $\mathcal{QUINTESSENCE}$-like → $\eta_\alpha = \langle \partial_\alpha \Phi \rangle_{\underline{0}}$

- **Lorentz non-covariant CPT-even perturbations**

$$\vec{B}^{\,2} \quad \to \quad (1+\epsilon)\vec{B}^{\,2}$$

  - **velocity of light** $\quad v_{\text{light}} \;=\; c\,\sqrt{1+\epsilon} \;\neq\; c$
  - **maximal attainable velocity of a material body**

$$v_{\text{body}} \;\leq\; c$$

  - $v_{\text{light}} \;<\; v_{\text{body}} \qquad \to$ **vacuum** *Čerenkov* **radiation**

  primary cosmic p up to 10^{20} eV $\quad\Rightarrow\quad |\epsilon| \;<\; 10^{-22}$

- **Evading the** *GZK* **Cutoff ?** $\sim 10^{20}$ eV from ~ 50 Mpc

  `K. Greisen; G. Zatsepin & V. Kuz'min (1966)`

  - **Dispersion relation** of the a -type particle

$$E_a = \sqrt{c_a^2 \vec{p}^{\,2} + m_a^2 c_a^4} \quad\Rightarrow\quad v_a \;\leq\; c_a$$

  - **Cosmic Microwave Photon scattering**

$$p + \gamma_{\text{CMB}} \;\to\; \Delta(1232) \qquad\qquad p + \gamma_{\text{CMB}} \;\to\; p + \pi$$

<u>**NO *GZK* Cutoff**</u> $\quad\Leftrightarrow\quad c_\Delta > c_p > c_\pi$

1. **MAXWELL CHERN SIMONS** FREE PHOTONS

- **MCS free Lagrange density in the Axial Gauge**

$$\mathcal{L}_{\mathrm{MCS}} = -\frac{1}{4}F^{\nu\lambda}(x)F_{\nu\lambda}(x) - \frac{1}{2}\eta_\alpha A_\beta(x)\tilde{F}^{\alpha\beta}(x) - B(x)\eta_\alpha A^\alpha(x)$$

  - $\tilde{F}^{\alpha\beta}(x) \equiv (1/2)\epsilon^{\alpha\beta\rho\sigma}F_{\rho\sigma}(x)$
  - $B(x)$ is the auxiliary field
  - η_α **Chern-Simons** & Axial Gauge **constant** vector

- Going in the momentum representation

$$A_\nu(x) \equiv \int \frac{d^4k}{(2\pi)^{3/2}}\tilde{A}^\nu(k)\exp\{ik\cdot x\}$$

- Euler Lagrange equations of motion

$$\left(k^2 g^{\nu\sigma} - i\epsilon^{\mu\rho\nu\sigma}\eta_\mu k_\rho\right)\tilde{A}_\sigma \;=\; 0$$

$$\eta^\sigma\tilde{A}_\sigma(k) \;=\; k^\sigma\tilde{A}_\sigma(k) \;=\; 0$$

- **MCS polarizations: 2D projector $\perp$ to η^ν and k^μ**

$$\mathrm{e}^{\mu\nu} \equiv g^{\mu\nu} - \frac{\eta\cdot k}{\mathtt{D}}\left(\eta^\mu k^\nu + \eta^\nu k^\mu\right) + \frac{k^2}{\mathtt{D}}\eta^\mu\eta^\nu + \frac{\eta^2}{\mathtt{D}}k^\mu k^\nu$$

$$\mathtt{D} \equiv (\eta\cdot k)^2 - \eta^2 k^2$$

$$\mathrm{e}^{\mu\nu}\eta_\nu = \mathrm{e}^{\mu\nu}k_\nu = 0 \qquad \mathrm{e}^{\mu}{}_{\lambda}\mathrm{e}^{\lambda\nu} = \mathrm{e}^{\mu\nu} \qquad \mathrm{e}^{\mu}{}_{\mu} = 2$$

- **Linear polarization real vectors**

$$\mathrm{e}_{\mu\nu} = -\sum_{a=1,2}\mathrm{e}_\mu^{(a)}\mathrm{e}_\nu^{(a)} \qquad g^{\mu\nu}\mathrm{e}_\mu^{(a)}\mathrm{e}_\nu^{(b)} = -\,\delta^{ab}$$

- **Chiral polarization complex vectors**

$$\varepsilon_\mu^{(L)} \equiv \frac{1}{2}\left(\mathrm{e}_\mu^{(1)} + i\mathrm{e}_\mu^{(2)}\right)$$

$$\varepsilon_\mu^{(R)} \equiv \frac{1}{2}\left(\mathrm{e}_\mu^{(1)} - i\mathrm{e}_\mu^{(2)}\right)$$

$$\mathrm{e}_{\mu\nu} = -\sum_{a=L,R}\left\{\varepsilon_\mu^{(a)}\varepsilon_\nu^{(a)*} + \varepsilon_\nu^{(a)}\varepsilon_\mu^{(a)*}\right\}$$

ONLY CHIRAL POLARIZATIONS ARE THE

EIGENSTATES OF THE **MCS** KINETIC OPERATOR

$$\left(k^2 g_\nu{}^\sigma - i\epsilon_\nu{}^{\mu\rho\sigma}\eta_\mu k_\rho\right)\varepsilon_\sigma^{(L)} = \left(k^2 - \sqrt{\mathtt{D}}\right)\varepsilon_\nu^{(L)}$$

$$\left(k^2 g_\nu{}^\sigma - i\epsilon_\nu{}^{\mu\rho\sigma}\eta_\mu k_\rho\right)\varepsilon_\sigma^{(R)} = \left(k^2 + \sqrt{\mathtt{D}}\right)\varepsilon_\nu^{(R)}$$

$$\Updownarrow$$

LET US SEARCH **FOUR REAL** SOLUTIONS

OF THE QUARTIC ON SHELL CONDITION

$$(k^2)^2 - \mathtt{D} = (k^2)^2 - (\eta \cdot k)^2 + \eta^2 k^2 = 0$$

MCS CHIRAL POLARIZATIONS

DO NOT EXACTLY COINCIDE WITH

MAXWELL'S CIRCULAR POLARIZATIONS

- **pure space-like** general solution $\eta_\mu = (0, \vec{\eta}\ )$

$$k_{0\pm}^2 = \vec{k}^{\ 2} + \frac{1}{2}\vec{\eta}^{\ 2} \pm |\vec{\eta}\ |\sqrt{\vec{k}^{\ 2}\cos^2\varphi + \frac{1}{4}\vec{\eta}^{\ 2}} \equiv \omega_\pm^2(\vec{k}, \vec{\eta}\ )$$

LEFT (+) and RIGHT (−) *deformed hyperboloyds*

$$\cos\varphi \equiv \frac{\vec{k}\cdot\vec{\eta}}{|\vec{k}\ ||\vec{\eta}\ |}$$

$$\tilde{A}_\mu(k) = \sum_{a=L,R} \varepsilon_\mu^{(a)}(k) F_a(k)$$

$$F_L(k) = f_L(\vec{k})\delta[k_0^2 - {\omega_+}^2(\vec{k}, \vec{\eta})]$$

$$F_R(k) = f_R(\vec{k})\delta[k_0^2 - {\omega_-}^2(\vec{k}, \vec{\eta})]$$

$f_L(\vec{k})$ and $f_R(\vec{k})$ are **regular functions** on the support of the δ distributions (*deformed hyperboloyds*)

monochromatic plane wave solutions

are possible **only with** a

definite chiral polarization

$$A^{(L)}_{\mu,\vec{k}}(t,\vec{x}) = f_L \varepsilon^{(L)}_{\mu} \exp\left\{ i\vec{k}\cdot\vec{x} - it\omega_{+}(\vec{k},\vec{\eta}) \right\} \ + \ \text{c.c.}$$

$$A^{(R)}_{\mu,\vec{k}}(t,\vec{x}) = f_R \varepsilon^{(R)}_{\mu} \exp\left\{ i\vec{k}\cdot\vec{x} - it\omega_{-}(\vec{k},\vec{\eta}) \right\} \ + \ \text{c.c.}$$

WAVE PACKETS ARE SLOWLY SPLITTED INTO

LEFT (+) AND RIGHT (−)

WAVE PACKETS MOVING WITH

DIFFERENT GROUP VELOCITIES

$$\vec{v}_{\pm} = \frac{\vec{k}}{\omega_{\pm}(\vec{k},\vec{\eta})} \pm \frac{\vec{\eta}\ \vec{k}\cdot\vec{\eta}}{\omega_{\pm}(\vec{k},\vec{\eta})\sqrt{\left(2\vec{k}\cdot\vec{\eta}\right)^2 + |\vec{\eta}\ |^4}}$$

$$|\vec{v}_{\pm}| < 1$$

* VACUUM BIREFRINGENCE *

Group Velocity Difference between

Left and Right Wave Packets

$$\Delta v \equiv |\vec{v}_R - \vec{v}_L|$$

- **Vanishes at HIGH frequencies**

$$\lim_{|\vec{k}|\to\infty} \Delta v = 0$$

- **Grows at LOW frequencies**

$$\lim_{|\vec{k}|\to 0} \Delta v = 1$$

TIME DELAY of LEFT HANDED WAVES

WITH RESPECT TO RIGHT HANDED ONES

IS CALLED **VACUUM BIREFRINGENCE**

*** UPPER BOUND ***

EXPERIMENTAL ABSENCE OF THIS EFFECT IN

RADIO-ASTRONOMY OBSERVATIONS OF

DISTANT **QUASARS** AND **RADIO GALAXIES**

$$\Updownarrow$$

$$|\vec{\eta}\,| \ \leq \ 10^{-33}\ \mathrm{eV}$$

`B. Nodland & J. P. Ralston (1997)`

`J. F. C. Wardle, R. A. Perley & M. E. Cohen (1997)`

- **pure time-like** general solution $\eta_\mu = (\eta_0, 0)$

$$\omega_\pm(\vec{k}, \eta_0) = \sqrt{|\vec{k}\,|(|\vec{k}\,| \pm |\eta_0|)}$$

WAVE PACKETS ARE **SUPERLUMINAL**

LEFT (+) AND RIGHT (−)

WAVE PACKETS MOVE WITH

TACHIONIC GROUP VELOCITIES

$$\vec{v}_\pm = \frac{\vec{k}}{\omega_\pm(\vec{k}, \eta_0)} \left(1 \pm \frac{|\eta_0|}{2|\vec{k}\,|}\right)$$

$$|\vec{v}_\pm| > 1$$

$\omega_-(\vec{k}, \eta_0)$ becomes **IMAGINARY** when $|\vec{k}\,| < |\eta_0|$

UNPHYSICAL SOLUTIONS

2. **CPT ODD FREE** DIRAC SPINOR FIELDS

- **CPT ODD free Lagrange density of Dirac spinors**

$$\mathcal{L}_{\text{spinor}} = \bar{\psi}(x)\,(i\gamma^\mu\partial_\mu - m - \gamma^\mu b_\mu\gamma_5)\,\psi(x)$$

◦ b_μ **constant** axial-vector

◦ $\bar{\psi}(x)\gamma^\mu b_\mu\gamma_5\psi(x)$ breaks **Lorentz** and **CPT**

- momentum space **CPT ODD** Dirac equation

$$\psi(x) \equiv \int \frac{d^4p}{(2\pi)^{3/2}}\ \tilde{\psi}(p)\ \exp\{ip\cdot x\}$$

$$(\gamma^\mu p_\mu - m - \gamma^\mu b_\mu\gamma_5)\,\tilde{\psi}(p)\ =\ 0$$

○ **Diagonalization** of Dirac's operator

$$\tilde{\psi}(p) \;\equiv\; \mathrm{A}\;\mathrm{B}\;\tilde{\phi}(p)$$

$$\mathrm{A} \equiv p^2 + b^2 - m^2 + 2\left(b\cdot p + m b_\mu \gamma^\mu\right)\gamma_5$$

$$\mathrm{B} \equiv \gamma^\nu p_\nu + m + b_\nu \gamma^\nu \gamma_5$$

$\star$ **quartic scalar free field equation** $\star$

$$\left[\left(p^2 + b^2 - m^2\right)^2 + 4b^2m^2 - 4(b\cdot p)^2\right]\tilde{\phi}(p) = 0$$

$\star$ **on shell condition for CPT odd spinors** $\star$

$$\left(p^2 + b^2 - m^2\right)^2 + 4b^2m^2 - 4(b\cdot p)^2 = 0$$

- **pure space-like** general solution $b_\mu = (0, \vec{b}\ )$

$$p_{0\pm}^2 = \vec{p}^{\ 2} + \vec{b}^{\ 2} + m^2 \pm 2|\vec{b}\ |\sqrt{\vec{p}^{\ 2}\cos^2\theta + m^2}$$

$$\cos\theta \equiv \frac{\vec{p}\cdot\vec{b}}{|\vec{p}\ ||\vec{b}\ |}$$

- **pure time-like** general solution $b_\mu = (b_0, 0)$

$$p_{0\pm}^2 = (|\vec{p}\ | \pm b_0)^2 + m^2$$

$(+)$ **POSITIVE HELICITY particles**

NEGATIVE HELICITY anti-particles

$(-)$ **NEGATIVE HELICITY particles**

POSITIVE HELICITY anti-particles

(+) **1P STATES**

- **pure space-like** case $\qquad b_\mu = (0, \vec{b}\,)$

$$
\begin{aligned}
p_+^2 &\equiv p_{0+}^2 - \vec{p}^{\,2} \\
&= \vec{b}^{\,2} + m^2 + 2|\vec{b}\,|\sqrt{\vec{p}^{\,2}\cos^2\theta + m^2} \;>\; 0
\end{aligned}
$$

- **pure time-like** case $\qquad b_\mu = (b_0, 0)$

$$
\begin{aligned}
p_+^2 &\equiv p_{0+}^2 - \vec{p}^{\,2} \\
&= {b_0}^2 + m^2 + 2|b_0||\vec{p}\,| \;>\; 0
\end{aligned}
$$

(+) **1P energy-momentum eigenstates**

$\star$ **PHYSICAL STATES** $\star$

$$
p_+^2 \;>\; 0 \qquad\qquad \forall\, \vec{p} \,\in\, \mathbf{R}^3
$$

$(-)$ 1P STATES

- **pure space-like** case $\qquad b_\mu = (0, \vec{b}\ )$

$$p_-^2 \ \equiv \ p_{0-}^2 - \vec{p}^{\ 2} \ > \ 0 \quad \Leftrightarrow \quad |\vec{p} \cdot \vec{b}\ | \ < \ \frac{1}{2}|\vec{b}^{\ 2} - m^2|$$

- **pure time-like** case $\qquad b_\mu = (b_0, 0)$

$$p_-^2 \ \equiv \ p_{0-}^2 - \vec{p}^{\ 2} \ > \ 0 \quad \Leftrightarrow \quad |\vec{p}\ | \ < \ \frac{{b_0}^2 + m^2}{2|b_0|}$$

$(-)$ 1P states separated by

THE PHYSICAL LIGHT-CONE BORDER

$$|p_{0-}| \ = \ |\vec{p}\ | \ = \ \frac{|b^2 + m^2|}{2|b_0 \mathrm{sgn}(p_0) - |\vec{b}\ | \cos\theta|} \qquad \forall b_\mu$$

$(-)$ **1P energy-momentum eigenstates**

$\star$ **PHYSICAL STATES** $\star$

$$p_-^2 \;\geq\; 0 \quad \Leftrightarrow \quad |\vec{p}\,| \;\leq\; \frac{|b^2+m^2|}{2|b_0\mathrm{sgn}(p_0)-|\vec{b}\,|\cos\theta|}$$

$(-)$ **1P energy-momentum eigenstates**

$\star$ **UNPHYSICAL STATES** $\star$

$$p_-^2 \;<\; 0 \quad \Leftrightarrow \quad |\vec{p}\,| \;>\; \frac{|b^2+m^2|}{2|b_0\mathrm{sgn}(p_0)-|\vec{b}\,|\cos\theta|}$$

$\star$ **GROUP VELOCITIES** $\star$

OF SPINOR WAVE PACKETS

$$|\vec{v}_\pm| < 1 \qquad \forall\ b^2\ >\ 0 \qquad \forall\ \vec{p}\ \in\ \mathbf{R}^3$$

$$|\vec{v}_\pm| < 1 \qquad b_\mu = (0, \vec{b}\ ) \qquad \forall\ \vec{p}\ \in\ \mathbf{R}^3$$

CPT ODD FREE SPINOR QUANTIZATION

$\star$ **CONSISTENT AND CAUSAL** $\star$

$$\forall\ b^2\ >\ 0$$

$$b_\mu = (0, \vec{b})$$

$\star$ $(-)$ **UNPHYSICAL STATES** $\star$

at very high momenta

⋆ EXPERIMENTAL LIMITS ⋆

- **spatial components $\vec{b}$ constrained by**

○ **PENNING TRAPS EXP**

`H. G. Dehmelt et al. (1999)` $|\vec{b}| < 5 \times 10^{-16}$ eV

○ **HYDROGEN MASERS EXP**

`D. F. Phillips et al. (2001)` $|\vec{b}| < 10^{-18}$ eV

○ **SPIN POLARIZED MATTER EXP**

`B. R. Heckel et al. (2002)` $|\vec{b}| < 10^{-20}$ eV

- **temporal component b_0 constrained by**

○ **PRECISION DETERMINATION OF m_e**

`Particle Data Group (2000)` $|b_0| < 10^{-2}$ eV

LORENTZ and CPT Breaking owing to

TINY b_0 NOT EXCLUDED AT PRESENT

3. RADIATIVELY INDUCED **CS** ACTION

- **CPT ODD QED Lagrange density**

$$\mathcal{L} \;=\; \mathcal{L}_{\rm MCS} \;+\; \mathcal{L}_{\rm spinor} \;+\; e\bar{\psi}(x)\gamma^{\mu}\psi(x)A_{\mu}(x)$$

$-|e|$ is the electron charge

$$\mathcal{L}_{\rm MCS} = -\frac{1}{4}F^{\nu\lambda}(x)F_{\nu\lambda}(x) - \frac{1}{2}\eta_{\alpha}A_{\beta}(x)\tilde{F}^{\alpha\beta}(x) - B(x)\eta_{\alpha}A^{\alpha}(x)$$

$$\mathcal{L}_{\rm spinor} = \bar{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m - \gamma^{\mu}b_{\mu}\gamma_5\right)\psi(x)$$

- **To be definite without loss of generality**

$$\eta_{\mu} = (0, \vec{\eta}\ ) \qquad\qquad b_{\mu} = (b_0 > 0, 0)$$

PHYSICAL 1P ASYMPTOTIC STATES

- **high momenta electron & positron decay**

$$e^-_{+-} \longrightarrow e^-_{+-} + e^-_- + e^+_+$$

$$e^+_{-+} \longrightarrow e^+_{-+} + e^-_- + e^+_+$$

V. Kostelecky & R. Lehnert (2001)

- **momentum conservation** $\vec{p} = \vec{k}_1 + \vec{k}_2 + \vec{k}_3$

$$\vec{k}_j = \beta_j \vec{p} + \vec{\Delta}_j \qquad j = 1, 2, 3$$

$$|\vec{\Delta}_j| \ll |\vec{p}| \qquad \vec{p} \cdot \vec{\Delta}_j = 0$$

$$\beta_1 + \beta_2 + \beta_3 = 1$$

$$\vec{\Delta}_1 + \vec{\Delta}_2 + \vec{\Delta}_3 = 0$$

- **helicity conservation** $|\vec{p}| \gg m$

- **energy conservation** $\qquad p_{0\pm} = k^{(1)}_{0\pm} + k^{(2)}_{0-} + k^{(3)}_{0-}$

$$\sqrt{(|\vec{p}\,| \pm b_0)^2 + m^2} \; =$$

$$\sqrt{\left(|\vec{k}_1| \pm b_0\right)^2 + m^2} \; +$$

$$\sqrt{\left(|\vec{k}_2| - b_0\right)^2 + m^2} \; + \; \sqrt{\left(|\vec{k}_3| - b_0\right)^2 + m^2}$$

decay processes take place

$$\Updownarrow$$

$$|\vec{p}\,| \; \geq \; \frac{2m^2}{b_0} \; \equiv \; \Lambda_s$$

.

$\star$ **STABLE 1P ASYMPTOTIC STATES** $\star$

$$p_{0\pm} \;=\; \sqrt{(|\vec{p}\,| \pm b_0)^2 + m^2} \qquad\qquad |\vec{p}\,| \;\leq \Lambda_s$$

$\star$ **PHYSICAL UV CUTOFF** Λ_s $\star$

$\star$ **PHYSICAL IN/OUT 1P STATES** $\star$

$$p_{\pm}^2 \;\geq\; 0 \qquad\qquad |\vec{p}\,| \;\leq \Lambda_s$$

- **1Loop induced MCS polarization tensor**

$$\Pi^{\mu\nu}(k) = \int \frac{d^4p}{i(2\pi)^4} \mathtt{tr} \left\{ \gamma^\mu S(p) \gamma^\nu S(p-k) \right\}$$

- **Feynman's propagator of the spinor field**

$$S(p) = \frac{i \text{ A B}}{\left(p^2 + b^2 - m^2 + i\varepsilon\right)^2 - 4\left[(b \cdot p)^2 - m^2 b^2\right]}$$

$$\text{A} \equiv p^2 + b^2 - m^2 + 2\left(b \cdot p + m b_\mu \gamma^\mu\right)\gamma_5$$

$$\text{B} \equiv \gamma^\nu p_\nu + m + b_\nu \gamma^\nu \gamma_5$$

- **UV divergencies $\longrightarrow$ REGULARIZATION**

$$\mathtt{reg}\Pi^{\mu\nu}(k) = \mathtt{reg}\Pi^{\mu\nu}_{\text{even}}(k) + \mathtt{reg}\Pi^{\mu\nu}_{\text{odd}}(k)$$

- **physical UV cutoff**

$$\mathtt{reg}\Pi^{\mu\nu}(k) \;=\; \lim_{\Lambda_s\to\infty}$$

$$\int \frac{d\vec{p}}{i(2\pi)^3}\ \vartheta\left(\vec{p}^{\,2}-{\Lambda_s}^2\right)\int_{-\infty}^{+\infty}\frac{dp_0}{2\pi}\ \mathtt{tr}\left\{\gamma^\mu S(p)\gamma^\nu S(p-k)\right\}$$

- **dimensional regularization**

$$\mathtt{reg}\Pi^{\mu\nu}(k) \;=\; \lim_{\omega\uparrow 2}$$

$$\int \frac{d^{2\omega}p}{i(2\pi)^{2\omega}}\mathtt{tr}\left\{\gamma^\mu S(p)\gamma^\nu S(p-k)\right\}$$

BOTH REGULARIZATIONS GUARANTEE

<u>*MINIMAL*</u> **LORENTZ SYMMETRY BREAKING**

OWING ONLY TO $b_\mu = (b_0, 0)$

- **1Loop induced parity odd polarization tensor**

$$\texttt{reg}\Pi^{\mu\nu}_{\rm odd}(k;b,m) \;=\; 4\epsilon^{\mu\nu\rho\sigma}b_\rho k_\sigma \;\texttt{reg}\Pi_{\rm odd}(k;b,m)$$

INDUCED PARITY ODD EFFECTIVE ACTION

BOTH REGULATORS GIVE THE SAME RESULT

$$\begin{aligned}\texttt{reg}\Pi^{\mu\nu}_{\rm odd}(k;b,m) \;&\equiv\; 4\epsilon^{\mu\nu\rho\sigma}b_\rho k_\sigma \Pi_{\rm odd}(k=0;m^2/b^2)\\ =\;& \frac{i}{2\pi^2}\epsilon^{\mu\nu\rho\sigma}b_\rho k_\sigma\left\{1-\vartheta(-b^2-m^2)\sqrt{1+\frac{m^2}{b^2}}\right\}\end{aligned}$$

INDUCED CHERN-SIMONS LAGRANGE DENSITY

$$\Delta\mathcal{L}_{\rm CS} \;=\; -\,\frac{1}{2}(\Delta\eta_\alpha)A_\beta\tilde{F}^{\alpha\beta}$$

$$\Delta\eta_\alpha \equiv \frac{e^2}{2\pi^2}\;b_\alpha\left\{1-\vartheta(-b^2-m^2)\sqrt{1+\frac{m^2}{b^2}}\right\}$$

- **comparison with previous different results**

  R. Jackiw & V.A. Kostelecky (1999) M. Peréz-Victoria (1999) J.M. Chung & P. Oh (1999) M. Chaichian, W.F. Chen & R. Gonzalez Felipe (2001) J.M. Chung & B.K. Chung (2001) O.A. Battistel & G. Dallabona (2001)

all those analyses contain

ILLEGAL MATHEMATICAL MANIPULATIONS

- **the vanishing result** $\Delta\eta_{\mu} = 0$

  S.R. Coleman & S.L. Glashow (1999)

  G. Bonneau (2001)

NON MINIMAL Lorentz symmetry breaking

regulators are tacitly assumed

4. CONCLUSIONS

- **CONSISTENT QUANTIZATION**

$$\eta_\mu = (0, \vec{\eta}\ ) \qquad b^2 > 0 \qquad b_\mu = (0, \vec{b}\ )$$

- **N-flavours induced Chern-Simons vector**

$$\Delta\eta_\mu = \frac{2\alpha}{\pi} \sum_{a=1}^{N} b_\mu{}^a$$

- **EXPERIMENTAL BOUNDS**

$$|\vec{b}\ | < 10^{-20}\ \mathrm{eV} \qquad |b_0\ | < 10^{-2}\ \mathrm{eV}$$

$$\vec{\eta}\ =\ \vec{\eta}^{\ (0)} + \Delta\vec{\eta} \qquad |\vec{\eta}\,|\ <\ 10^{-33}\ \mathrm{eV} \qquad |\Delta\vec{\eta}\,|\ <\ 10^{-21}\ \mathrm{eV}$$

$$\Updownarrow$$

- ◦ **fine tuning cancellation between** $\vec{\eta}^{\ (0)}$ **and** $\Delta\vec{\eta}$

- ◦ $b_0 \neq 0$ **not yet excluded empirically**